

What is the value of  $0.1 + 0.2 + 0.3 \times 0.4$ ?

We have

$$0.1 + 0.2 + 0.3 \times 0.4 = 0.3 + 0.12 = 0.42$$

My train was scheduled to leave at 17:40 and to arrive at 18:20. However, it started five minutes late and the journey then took 42 minutes. At what time did I arrive?

It started at 17:45.

$$17:45 + 20 \text{ min} = 18:05$$

$$18:05 + 22 \text{ min} = 18:27$$

What is the remainder when 354972 is divided by 7?

We have

$$354972 = 350000 + 4900 + 70 + 2$$

$$= 7 \times 50000 + 7 \times 700 + 7 \times 10 + 2$$

So the remainder is 2

Which of the following numbers is three less than a multiple of 5 and 3 more than a multiple of 6?

A) 12 B) 17 C) 21 D) 22 E) 27

We have

$$12 = 15 - 3 = 9 + 3 \quad \times$$

$$17 = 20 - 3 = 14 + 3 \quad \times$$

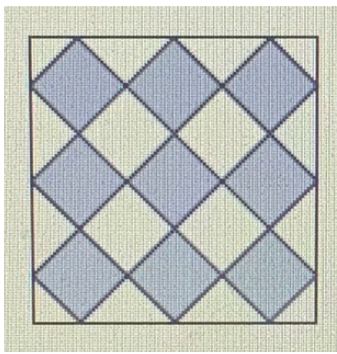
$$21 = 24 - 3 \quad \times$$

$$22 = 25 - 3 = 19 + 3 \quad \times$$

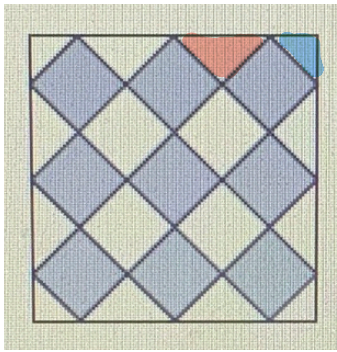
$$27 = 30 - 3 = 24 + 3 \quad \checkmark$$

So the answer is 27

In the diagram, the small squares are all the same size. What Fraction of the large square is shaded?



The red triangle has half the area of the shaded square. There are 8 triangles congruent to it in the diagram. This covers an area equivalent to 4 squares.



The blue triangle has a quarter of the area of a shaded square. There are 4 such triangles in the diagram.

There are 9 shaded squares. The unshaded area is equivalent to

$$4 + 4 + 1 = 9$$

shaded squares. So the Fraction of the diagram which is shaded is  $\frac{1}{2}$

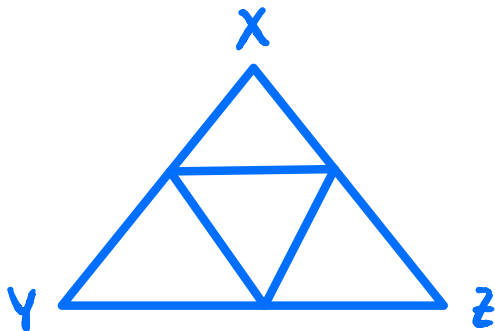
When the following Fractions are put in their correct places on the number line, which Fraction is in the middle?

$$-\frac{1}{7} \quad \frac{1}{6} \quad -\frac{1}{5} \quad \frac{1}{4} \quad -\frac{1}{3}$$

We have

$$-\frac{1}{3} < -\frac{1}{5} < -\frac{1}{7} < \frac{1}{6} < \frac{1}{4}$$

The equilateral triangle  $XYZ$  is fixed in position. Two of the four small triangles are to be painted black. In how many different ways can this be done?





If we just choose from the corner triangles, there are  ${}^3C_2 = 3$  ways.

If we choose the middle triangle and a corner triangle, there are  ${}^3C_1 = 3$  ways.

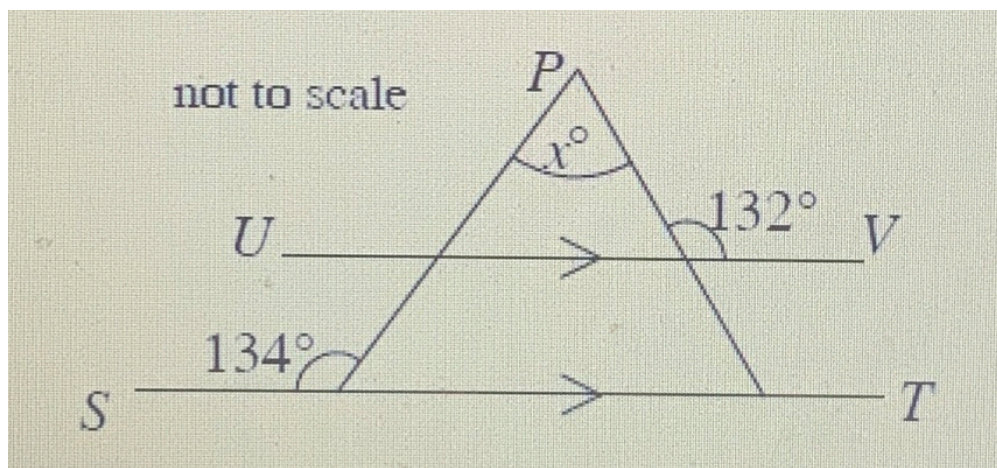
So in total there are  $3 + 3 = 6$

Amy, Ben and Chris are standing in a row. If Amy is to the left of Ben and Chris is to the right of Amy, which of these statements must be true?

- A) Ben is Furthest to the left
- B) Chris is Furthest to the right
- C) Amy is in the middle
- D) Amy is Furthest to the left
- E) None of the above is true

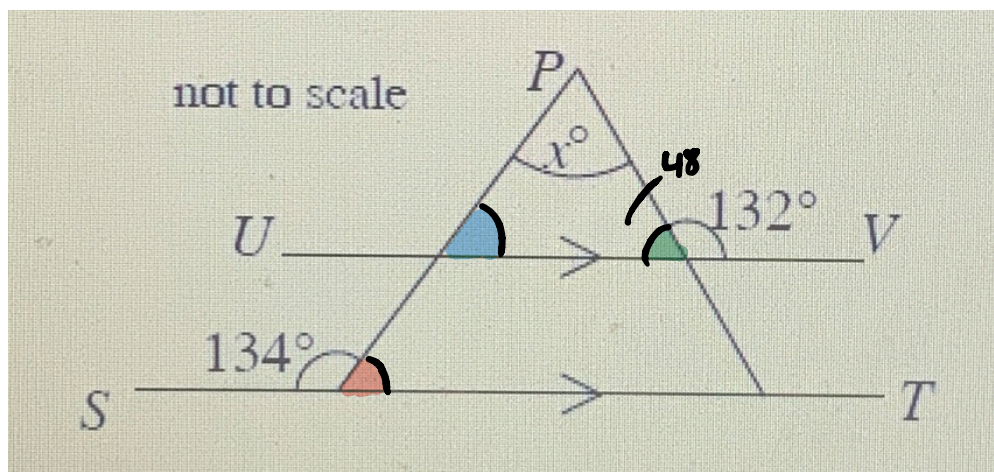
Since Chris is to the right of Amy, she is to the left of him. So **D** is true

In the diagram,  $ST$  is parallel to  $UV$ . Find  $x$



By supplementary angles, the green angle is  $180 - 132 = 48^\circ$  and the red angle is  $180 - 134 = 46^\circ$ .

By corresponding angles, the blue angle is also  $46^\circ$  (same as the red angle)



By angle sum of a triangle

$$x = 180 - 48 - 46$$

$$= 86^\circ$$

Which of the following has the largest value?

A)  $\frac{1}{2} + \frac{1}{4}$     B)  $\frac{1}{2} - \frac{1}{4}$     C)  $\frac{1}{2} \times \frac{1}{4}$     D)  $\frac{1}{2} \div \frac{1}{4}$     E)  $\frac{1}{4} \div \frac{1}{2}$

We have

$$A = \frac{3}{4}$$

$$B = \frac{1}{4}$$

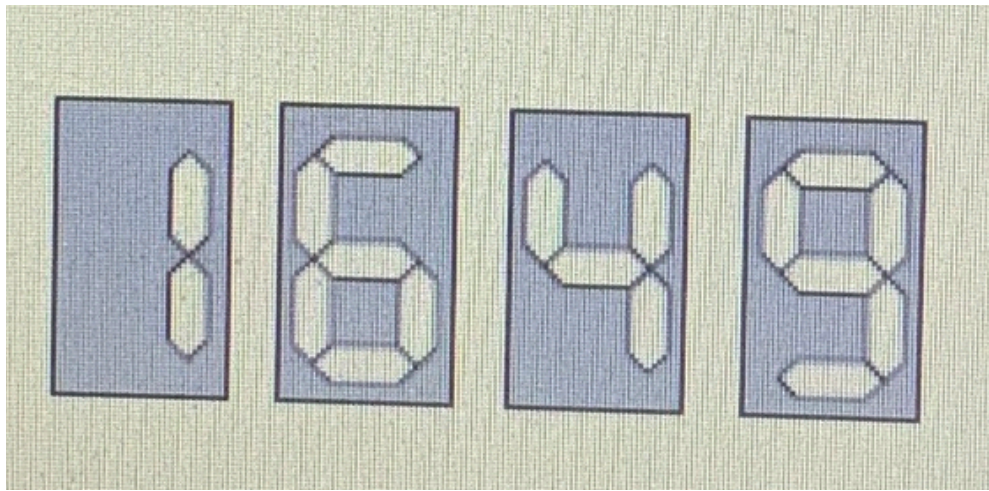
$$C = \frac{1}{8}$$

$$D = \frac{1}{2} \times \frac{4}{1} = 2$$

$$E = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$$

So the answer is **D**

A station clock shows each digit by illuminating up to 7 bars in a display. For example, the displays for 1, 6, 4, 9 are shown. When all the digits from 0 to 9 are shown in turn, which bar is used least?



0 1 2 3 4 5 6  
7 8 9

So the answer is **lower left**

| Tally       |  |
|-------------|--|
| Top         |  |
| Upper left  |  |
| Lower left  |  |
| Bottom      |  |
| Lower right |  |
| Upper right |  |
| Middle      |  |

The six-member squad For the ladybirds S-a-side team consists of a 2-spot ladybird, a 10-spot ladybird, a 14-spot, 18-spot, 24-spot and a pine ladybird (on the bench). The average number of spots for members of the squad is 12. How many spots has the pine ladybird?

Suppose the pine ladybird has  $n$  spots. Then

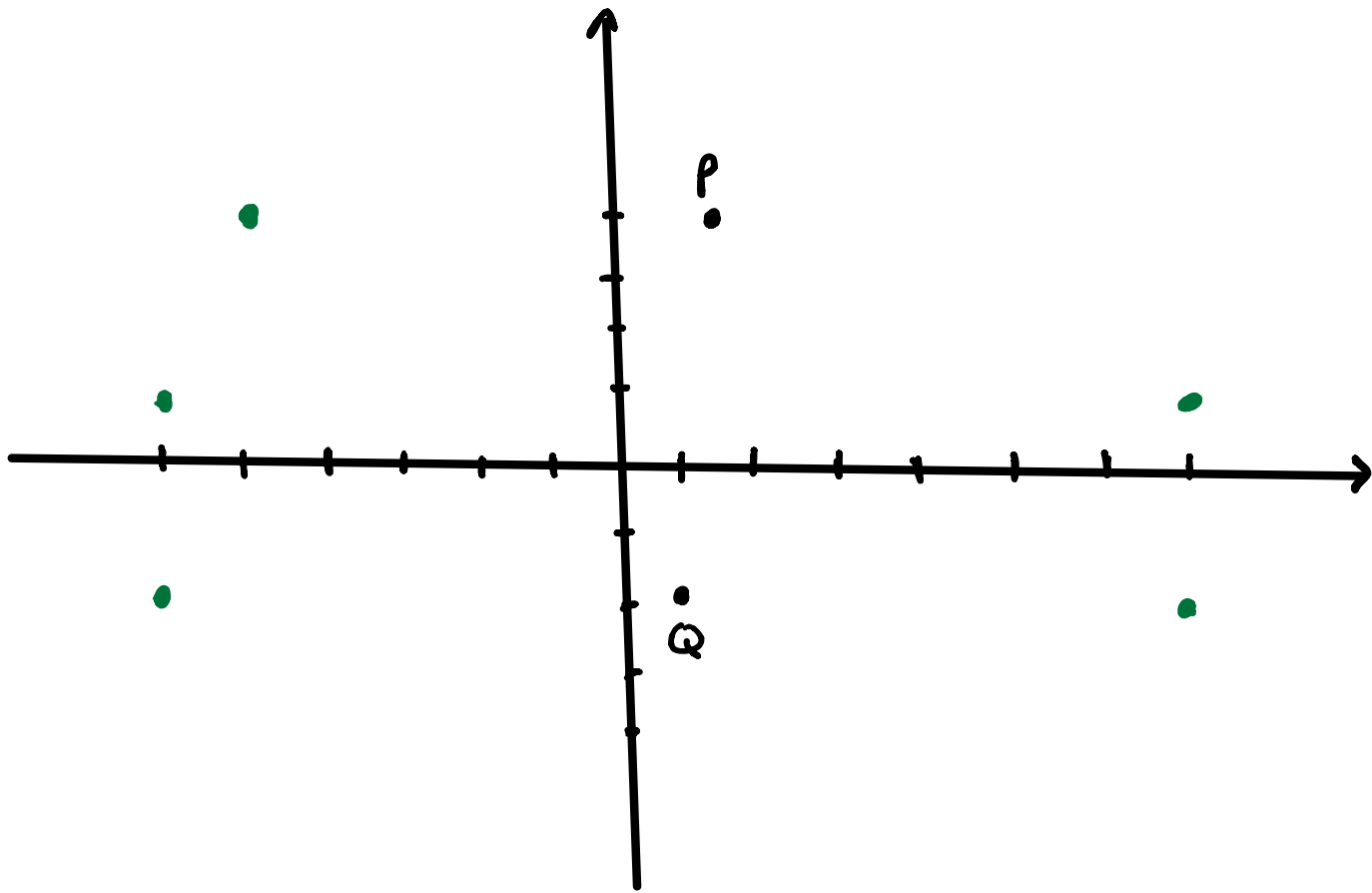
$$\frac{2 + 10 + 14 + 18 + 24 + n}{6} = 12$$

$$\Rightarrow n + 68 = 72$$

$$\Rightarrow n = 4$$

Points P and Q have coordinates (1,4) and (1,-2) respectively. For which of the following possible coordinates

of the point R would triangle  $\triangle PQR$  not be isosceles?  
A) (-5, 4)    B) (7, 1)    C) (-6, 1)    D) (-6, -2)    E) (7, -2)



Guess  $(-6, -2)$  based on diagram.

Check:

$$d(P, Q) = 6$$

$$d(P, (-6, -2)) = \sqrt{(1+6)^2 + (4+2)^2}$$

$$= \sqrt{49 + 36}$$

$$= \sqrt{85}$$

$$d(Q, (-6, -2)) = 7$$

So the answer is  $(-6, -2)$

A line is 0.2 mm thick. How long would it have to

be to cover an area of one square metre?

We have

$$\begin{aligned} 0.2 \text{ mm} &= 2 \times 10^{-1} \text{ mm} \\ &= 2 \times 10^{-2} \text{ cm} \\ &= 2 \times 10^{-4} \text{ m} \end{aligned}$$

So the length would have to be

$$\begin{aligned} \frac{1}{2 \times 10^{-4}} &= \frac{1}{2} \times 10^4 \\ &= 5000 \text{ m} \end{aligned}$$

I choose three numbers from this number square, including one number from each row and one number from each column. I then multiply the three numbers together. What is the largest possible product?

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

Case: 1 chosen from top row

$$1 \times 5 \times 9 = 45, \quad 1 \times 6 \times 8 = 48$$

Case: 2 chosen from top row

$$2 \times 4 \times 9 = 72, \quad 2 \times 6 \times 7 = 84$$

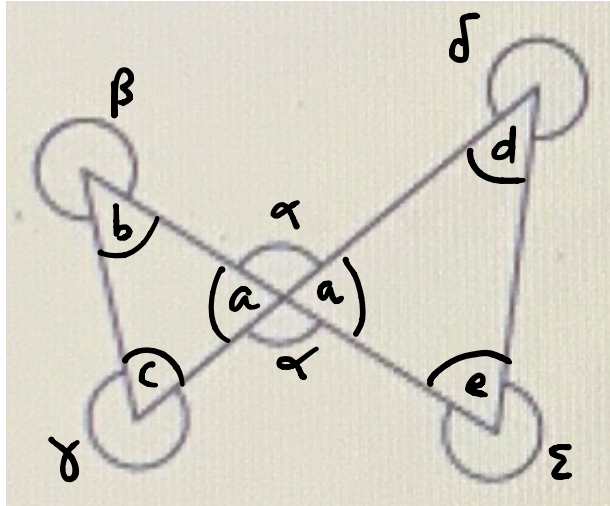
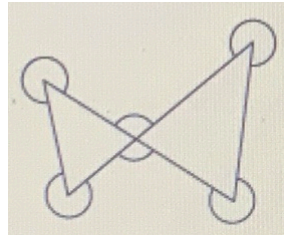
Case: 3 chosen from top row

$$3 \times 5 \times 7 = 105, \quad 3 \times 4 \times 8 = 96$$

So the answer is 105



What is the sum of the six marked angles?



Label the angles as shown.  
We have

$$a + b + c = a + d + e = 180$$

$$\alpha = 180 - a$$

$$\beta = 360 - b$$

$$\gamma = 360 - c$$

$$\delta = 360 - d$$

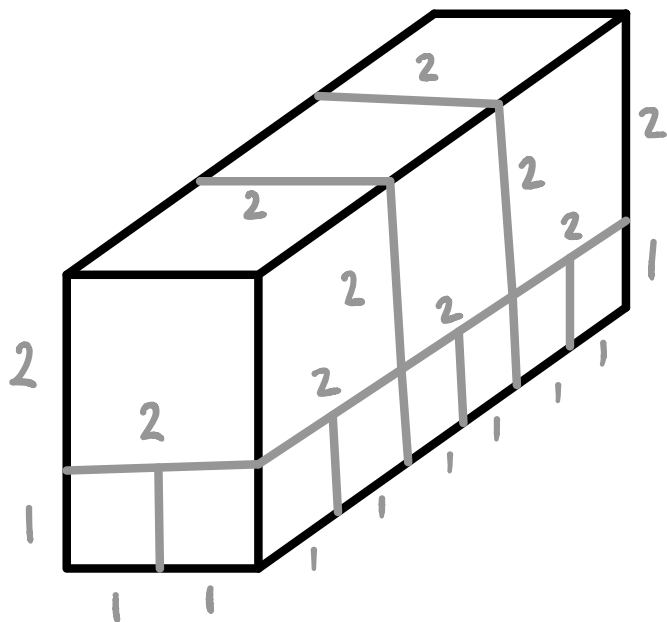
$$\epsilon = 360 - e$$

$$\begin{aligned} \text{So } 2\alpha + \beta + \gamma + \delta + \epsilon &= 2(180 - a) + 360 \times 4 - (b + c + d + e) \\ &= 360 \times 5 - (a + b + c) - (a + d + e) \\ &= 360 \times 5 - 180 - 180 \\ &= 360 \times 4 \\ &= 1440^\circ \end{aligned}$$

A rectangular block of fudge measures 2 inches by 3 inches by 6 inches. I want to cut the block into cubes whose side lengths are whole numbers of inches. What is the smallest number of cubes I can obtain?

The longest possible side length for a cube is 2 inches. To minimise the number of cubes, maximise the size of each cube. We can create three  $2 \times 2 \times 2$  cubes as shown, and the rest will be

$1 \times 1 \times 1$ .



So there will be 3  $2 \times 2 \times 2$  cubes and 12  $1 \times 1 \times 1$  cubes for a total of **15**

The letters J, M, C represent three different non-zero digits. What is  $J + M + C$ ?

$$\begin{array}{r} JJ \\ MM \\ CC \\ \hline JMC \end{array} +$$

We have

$$11J + 11M + 11C = 100J + 10M + C$$

$$\Rightarrow M + 10C = 89J$$

The maximum value of the left hand side is

$$8 + 10 \times 9 = 98$$

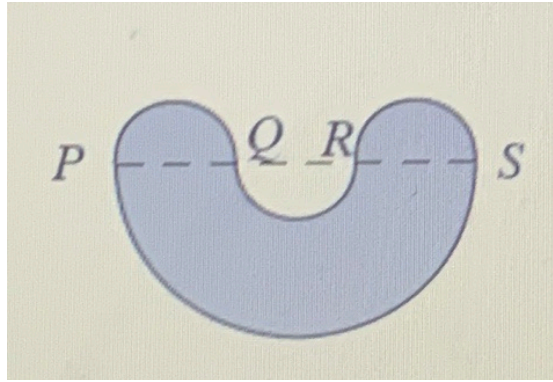
So the only possibility for the right hand side is

$J=1$ . In that case,  $M=9$  and  $C=8$  so

$$J + M + C = 1 + 9 + 8 = \mathbf{18}$$



The points P, Q, R, S lie in order along a straight line, with  $PQ = QR = RS = 2\text{ cm}$ . Semicircles with diameters PQ, QR, RS and SP join to make the shape shown. What is the area of the shape?



We have  $SP = 2 \times 3 = 6$ , so the area of the shape is equal to the area of a semicircle of radius 3 ( $= 6 \div 2$ ) plus the area of a semicircle of radius 1 ( $= 2 \div 2$ ). [Imagine moving one of the small semicircles to fill the hole in the larger one]

So the area is

$$\frac{1}{2} \cdot \pi \cdot 3^2 + \frac{1}{2} \cdot \pi \cdot 1^2 = 5\pi \text{ cm}^2$$

At half time, Boarwatts Academy had scored all of the points so far in their annual match against Range Hill School. In the second half, each side scored 3 points. At the end of the match, Boarwatts Academy had scored 90% of the points. What Fraction of the points in the match was scored in the second half?

Suppose that Boarwatts scored  $n$  points in the first half.

The total number of points in the match was

$$n + 3 + 3 = n + 6$$

Boarwarts scored  $n + 3$  points.

So

$$n + 3 = \frac{9}{10} (n + 6)$$

$$\Rightarrow 10(n + 3) = 9(n + 6)$$

$$\Rightarrow 10n + 30 = 9n + 54$$

$$\Rightarrow n = 24$$

Hence the Fraction of points scored in the second half was

$$\frac{6}{30} = \frac{1}{5}$$

A list of ten numbers contains two of each of the numbers 0, 1, 2, 3, 4. The two 0s are next to each other, the two 1's are separated by one number, the two 2s by two numbers, the two 3s by three numbers and the two 4s by four numbers. The list starts 3, 4, ... What is the last number?

Place the second 3 and 4 :

3 4   3  4

There are two ways we could place the 2's :

3 4   3  4

2

2

2

2

Case Purple placement

3 4 2   3 2 4

Now there is no number which can go in the red space, so this is impossible.

Case Green placement

3 4   3 2 4  2

We can complete the list:

3 4 0 0 3 2 4 1 2 1

So the last number is 1

Only one choice of the digit  $d$  gives a prime number for each of the 3-digit numbers read across and downwards in the diagram. Find  $d$

$$\begin{array}{c} 5 \\ 1 \, d \, 3 \\ 7 \end{array}$$

'5d7' has digit sum  $12+d$ , so  $d$  cannot be a multiple of 3 (i.e. not 0, 3, 6 or 9)

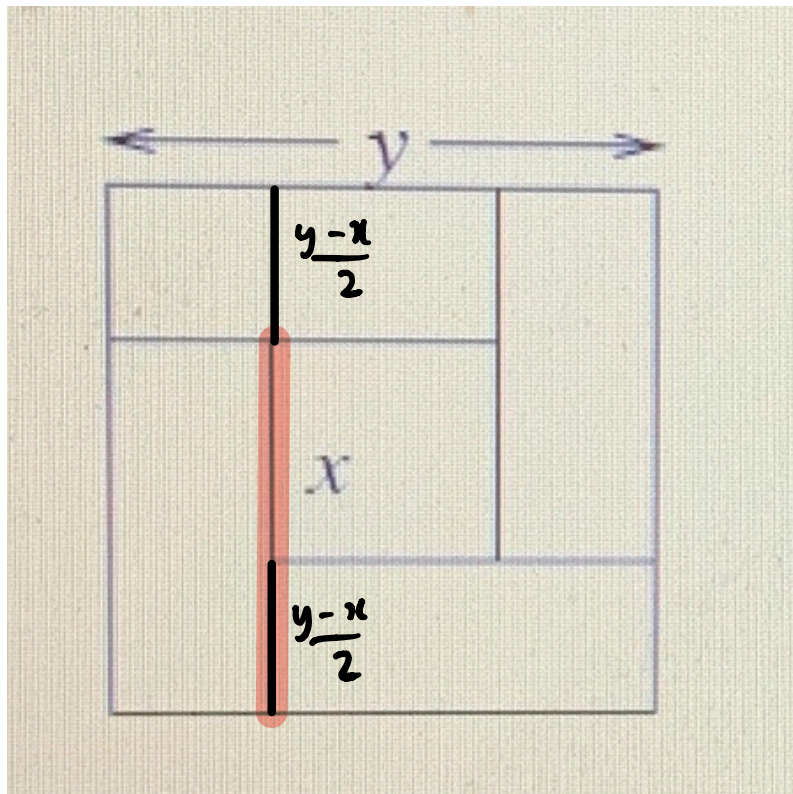
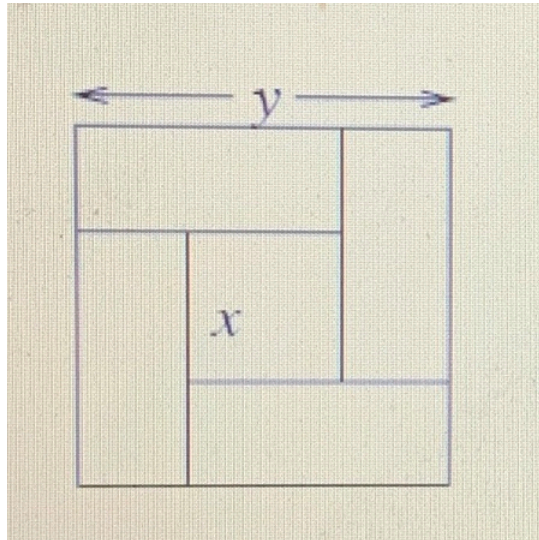
'1d3' has digit sum  $4+d$ , so  $d$  cannot be 2 more than a multiple of 3 (i.e. not 2, 5, 8)

If  $d=1$ , we have  $517 = 47 \times 11$  (not prime)

If  $d=4$ , we have  $143 = 13 \times 11$  (not prime)

So the only remaining option is  $d = \textcolor{red}{7}$

The diagram shows a square with sides of length  $y$  divided into a square with sides of length  $x$  and four congruent rectangles. What is the length of the longer side of each rectangle?



The short side of the rectangle (each of the black sections shown) has length

$$\frac{y-x}{2}$$

So the long side (highlighted) has length

$$\begin{aligned} x + \frac{y-x}{2} &= \frac{2x}{2} + \frac{y-x}{2} \\ &= \frac{x+y}{2} \end{aligned}$$

The pages of a book are numbered  $1, 2, 3, \dots$

In total, it takes 852 digits to number all the pages of the book. What is the number of the last page?

The pages 1 to 9 use 1 digit each for a total of 9.

The pages 10 to 99 use 2 digits each.

We have

$$10 \leq n \leq 99$$

$$\Rightarrow 1 \leq n-9 \leq 90$$

So there are 90 two-digit numbers. In total, they contribute  $90 \times 2 = 180$  digits.

Thus far, we have accounted for  $9 + 180 = 189$  digits.

Suppose that there are  $n$  3-digit numbers. They contribute a total of  $3n$  digits.

So the total number of digits is

$$189 + 3n = 852$$

$$\Rightarrow 3n = 663$$

$$\Rightarrow n = 221$$

We have

100  $\rightarrow$  1st 3-digit page

101  $\rightarrow$  2nd " "

$\vdots$

199  $\rightarrow$  100th " "

200  $\rightarrow$  101st " "

201  $\rightarrow$  102nd " "

$\vdots$

299  $\rightarrow$  200th " "

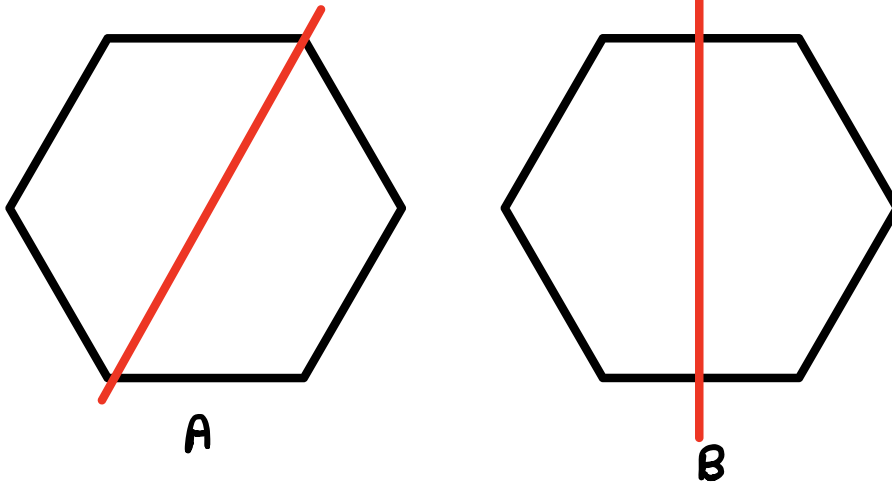
300  $\rightarrow$  201st " "

$\vdots$

320  $\rightarrow$  221st

A piece of paper in the shape of a polygon is folded in half along a line of symmetry. The resulting shape is also folded in half, again along a line of symmetry. The final shape is a triangle. How many possibilities are there for the number of sides of the original polygon?

Suppose that a polygon has  $n$  sides. If  $n$  is even, we can create a line of symmetry between 2 vertices or between the midpoints of two opposite sides:



In case A, after folding, the new polygon will have

$$\underbrace{\left(\frac{n-2}{2}\right)}_{\text{half the number of vertices not included in the fold}} + 2 = \frac{n}{2} - 1 + 2 = \frac{n}{2} + 1 \text{ vertices}$$

↑ the two vertices on the fold are included in both of the polygons created

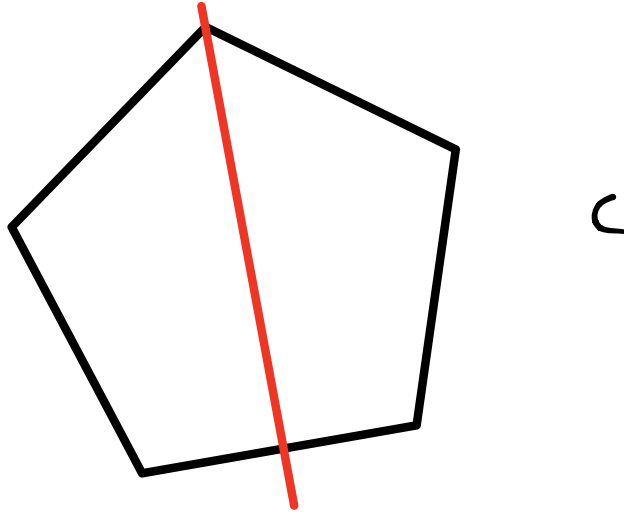
In case B, after folding, the new polygon will have

$$\frac{n}{2} + 2 \text{ vertices}$$

(half the vertices of the original polygon, plus two new ones are created)



If  $n$  is odd, we create a line of symmetry between one vertex and the middle of the opposite edge:



After folding, the number of vertices in the new polygon is

$$\frac{n-1}{2} + 2 = \frac{n+3}{2}$$

half the number of vertices not included in the fold      the one original vertex and the newly created vertex involved in the fold

Work backwards: suppose the original polygon has  $n$  edges and the second polygon has  $m$  edges. The final polygon has 3 edges.

If the final fold was type A, then

$$3 = \frac{m}{2} + 1$$

$$\Rightarrow \frac{m}{2} = 2$$

$$\Rightarrow m = 4$$

First fold A :  $4 = \frac{n}{2} + 1$

$$\Rightarrow 3 = \frac{n}{2}$$

$$\Rightarrow n = 6$$

First Fold B :  $4 = \frac{n}{2} + 2$

$$\Rightarrow 2 = \frac{n}{2}$$

$$\Rightarrow n = 4$$

First Fold C :  $4 = \frac{n+3}{2}$

$$\Rightarrow 8 = n+3$$

$$\Rightarrow n = 5$$

If the Final fold was type B then

$$3 = \frac{m}{2} + 2$$

$$\Rightarrow 1 = \frac{m}{2}$$

$$\Rightarrow m = 2$$

which is impossible since a polygon must have at least 3 sides.

If the Final fold was type C then

$$3 = \frac{m+3}{2}$$

$$\Rightarrow 6 = m+3$$

$$\Rightarrow m = 3$$

First Fold A  $3 = \frac{n}{2} + 1$

$$\Rightarrow 2 = \frac{n}{2}$$

$$\Rightarrow n = 4$$

First fold B  $3 = \frac{n}{2} + 2$

$$\Rightarrow 1 = \frac{n}{2}$$

$$\Rightarrow n = 2 \text{ impossible}$$

First fold C  $3 = \frac{n+3}{2}$

$$\Rightarrow 6 = n+3$$

$$\Rightarrow n = 3$$

So we have the following possibilities:  $n=3, 4, 5, 6$

Hence the answer is **4**